Quintessential Cosmological Scenarios in the Relativistic Theory of Gravitation*

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Abstract

It is shown that the accelerated expansion of the universe in the framework of the relativistic theory of gravitation can be achieved by the introduction of the quintessential term in the energy-momentum tensor. The value of the minimum scaling factor and the modern observational data for the density and state parameters of the matter give the rough estimations for the maximum graviton mass and the maximum scaling factor. The former can be very low in the case of the primordial inflation and the latter can be extremely large for the scalar field model of the quintessence. In any case, the massive gravitons stop the second inflation and provide the closed cosmological scenario in the agreement with the causality principle inherent to the theory.

1 Introduction

The relativistic theory of gravitation (RTG) [1, 2, 3] disagrees with the Einstein's general relativity (GR) in the crucial point: it denies the total geometrization and considers the gravitation on the basis of the classical Faraday-Maxwell's field approach. This means that there is the topologically simple background spacetime of the Minkowski type, which can be restored in any situation. As a result, we can detach the physical content from the arbitrary geometrical game with co-ordinates. This converts the gravitation

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from the tensor-geometrical concept to the tensor-field one and puts it on the unified level with the another fields.

Formally, the RTG can be considered as the bi-metric theory of the gravitation [4, 5]. However, in the RTG the effective Riemannian spacetime produced by the gravitational field is essentially separated from the Minkowski background because the latter is presented in the field equations (see next section). Naturally, this transforms the solutions of the field equations and has the pronounced physical consequences. For example, the singularity disappears and the graviton acquires the nonzero mass. Nevertheless, the basic observational consequences of the RTG coincide with those in the GR (for instance, Mercury perihelion motion, time delay and spectral shift in the gravitational field, see [2]).

The application of the RTG for the cosmology produces some astonishing results, viz., in virtue of the field equations the Friedmann-Robertson-Walker cosmology admits only the flat global efficient Riemannian spacetime without an initial singularity and with an oscillating time behavior [2, 3]. The initial expansion is stimulated by the antigravitation, which is caused by the massive gravitons in the strong gravitational fields. The initial temperature is defined by the graviton mass and can be too low to create the undesirable relics (e.g. monopoles). So, the problems of the cosmological spacetime flatness, the source of the initial expansion, the cosmological singularity and the absence of the relics find in the RTG a natural solution. However, in this theory there are some disagreements with the modern observational data. As it is known, the latter suggests the accelerated expansion of the universe at present (see, for example, [6, 7]). But in the RTG the accelerated expansion is possible only during a very short stage of the initial evolution and the subsequent expansion has a definitely decelerated character.

As it is well known, the accelerated cosmological expansion in the framework of the GR can be obtained "by hand" due to an insertion of the so-called cosmological constant in the field equations (for a review see [8]). This constant can be considered as a part of the geometrical structure of the GR because it is a natural consequence of the variational principle [9]. Alternatively, it is possible to treat the cosmological constant as the vacuum zero-point energy. But in the both cases its value is too small and can not be attributed to any known physical scale.

The situation in the RTG is more complicated by virtue of the vacuum stability principle: the absence of the material fields reduces the effective Riemannian spacetime to the Minkowski one. Hence, the cosmological constant can not be introduced by hand and is to have the gravitational nature concerned with the nonzero graviton mass. As a result, the cosmological constant-like action of the massive graviton in the RTG produces the *deceleration* of the cosmological expansion.

Nevertheless there exists an approach, which considers the accelerated expansion of the universe as a manifestation of some matter possessing an unusual equation of the state $p = w\rho$ (where p is the pressure and ρ is the density). This matter usually is called as the X-matter or *quintessence*. If its state parameter w lies between the limits of the strong and week energy conditions (i.e. $-1 \leq w \leq -\frac{1}{3}$), the domination of such matter produces a repulsion causing the accelerated expansion of the universe [10]. The best candidate here is a certain scalar field whose potential energy dominates at present (the survey can be found in [11], for example).

In this article we shall consider the implementation of this idea in the RTG framework. As a result, some restrictions on the key parameter of the theory, i. e. the graviton mass, will be obtained.

2 Basic equations

The field equations for the gravitational field in the framework of the RTG is based on the assumption that the universal character of the gravitation allows to introduce the effective Riemannian spacetime [3]:

$$\widetilde{g}^{\mu\nu} = \widetilde{\gamma}^{\mu\nu} + \widetilde{\varrho}^{\mu\nu}, \tag{1}$$

where $\widetilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, $\widetilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu}$, $\widetilde{\varrho}^{\mu\nu} = \sqrt{-\gamma}\varrho^{\mu\nu}$ are the densities of Riemannian metric tensor, Minkowski metric tensor and gravitational field tensor, respectively. In this case the Lagrangian density for the gravitational field is the function both $\widetilde{\varrho}^{\mu\nu}$ and $\widetilde{\gamma}^{\mu\nu}$. It is essential that the effective Riemannian spacetime is completely defined for the given Minkowski coordinates, i. e. $g^{\mu\nu}$ is their single-valued function. Hence, the topology of the effective spacetime is quite simple. Let us consider the infinitesimal transformation of coordinates by means of the translation vector ζ^{μ}:

$$x^{\mu\prime} = x^{\mu} + \zeta^{\mu}. \tag{2}$$

Then the field-dependent metric density of the effective spacetime is changed as:

$$\widetilde{g}^{\mu\nu\prime} = \widetilde{g}^{\mu\nu} + \delta_{\zeta}\widetilde{g}^{\mu\nu} + \zeta^{\lambda} D_{\lambda}\widetilde{g}^{\mu\nu}, \tag{3}$$

δ_ζ is the Lie variation and D_λ is the covariant derivative on the Minkowski (i.e. background) spacetime. If the Lagrangian density for the gravitational field depends only on $\widetilde{g}^{\mu\nu}$ and its derivatives, then the transformation (3) changes this density only on a divergence. Bsing on (3) the crucial issue is the definition of the gauge group preserving the field equations and background metrics. Let's Eq. (3) describes the transformation produced by the infinite-dimensional gauge group with the gauge vector ζ^μ. In contrast to the coordinate transformation, this gauge transformation does not effect the background: $\delta_\zeta \widetilde{g}^{\mu\nu} = \delta_\zeta \widetilde{\varrho}^{\mu\nu}$.

The simplest Lagrangian density, which is changed only on a divergence by this gauge transformation, can be constructed from $\sqrt{-g}$ and $\widetilde{\Re} = \sqrt{-g}\Re$ ($\Re$ is the scalar curvature of the effective spacetime). Let us define (see [2, 3]) the scalar curvature density through the tensor $F^\mu_{\nu\lambda}$:

$$F^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\kappa} \left(D_\nu g_{\kappa\lambda} + D_\lambda g_{\kappa\nu} - D_\kappa g_{\nu\lambda} \right). \tag{4}$$

Then

$$\widetilde{\Re} = -\widetilde{g}^{\mu\nu} \left(F^\lambda_{\mu\nu} F^\kappa_{\lambda\kappa} - F^\lambda_{\mu\kappa} F^\kappa_{\nu\lambda} \right) - D_\nu \left(\widetilde{g}^{\mu\nu} F^\kappa_{\mu\kappa} - \widetilde{g}^{\mu\kappa} F^\nu_{\mu\kappa} \right). \tag{5}$$

Hence, the required density resulting in the field equation with the derivatives up to second order has the following form:

$$L_g = -\omega_1 \widetilde{g}^{\mu\nu} \left(F^\lambda_{\mu\nu} F^\kappa_{\lambda\kappa} - F^\lambda_{\mu\kappa} F^\kappa_{\nu\lambda} \right) + \omega_2 \sqrt{-g}, \tag{6}$$

where ω_1 and ω_2 are the some constants.

However, the structure of the Lagrangian density (6) does not allow to include the background metrics in the field equations. Therefore we have to add in Eq. (6) the terms explicitly containing $\gamma_{\mu\nu}$ and violating considered gauge group [3, 12]. The term $\gamma_{\mu\nu}\widetilde{g}^{\mu\nu}$ obeys the transformational properties under consideration but only for the gauge vectors:

$$g^{\mu\nu} D_\mu D_\nu \zeta^\lambda = 0. \tag{7}$$

Resulting Lagrangian density for the gravitational field is:

$$L_g' = -\omega_1 \widetilde{g}^{\mu\nu} \left(F^\lambda_{\mu\nu} F^\kappa_{\lambda\kappa} - F^\lambda_{\mu\kappa} F^\kappa_{\nu\lambda} \right) + \omega_2 \sqrt{-g} + \omega_3 \gamma_{\mu\nu} \widetilde{g}^{\mu\nu} + \omega_4 \sqrt{-\gamma}, \tag{8}$$

here the last term is introduced to provide the vacuum stability, i.e. to exclude the cosmological constant-like term in the absence of the matter.

From the variational principle for the gravitational field ($\frac{\delta L_g^{'}}{\delta \widetilde{g}^{\mu\nu}} = 0$), the vacuum stability requirement and taking into account the material sources for the gravitational field we can obtain from (8) the field equation:

$$G^\mu_\nu - \frac{m^2}{2}\left(\delta^\mu_\nu + g^{\mu\lambda}\gamma_{\lambda\nu} - \frac{1}{2}\delta^\mu_\nu g^{\kappa\lambda}\gamma_{\kappa\lambda}\right) = -\frac{8\pi\varkappa}{c^4}T^\mu_\nu, \tag{9}$$

where $m^2 = (m_g c/\hbar)^2$, $\varkappa$ is the Newtonian gravitational constant, m_g is the graviton mass as a natural interpretation of the constants ω incoming in the Lagrangian density, G^μ_ν is the Einstein tensor. Below we shall use $c = \hbar = 1$, then $m_{pl} = 1/\sqrt{8\pi\varkappa} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. It should be noted, that the mass of graviton results from the gauge group violation, i.e. it appears together with the background metrics in the Lagrangian. Otherwise we have the usual Einstein-Gilbert field equation (without cosmological constant) and the background spacetime loses its physical meaning.

Now let us consider the physical sense of the constraint (7). As a matter of fact, the introduced simplest modification of the Lagrangian by the term $\gamma_{\mu\nu}\widetilde{g}^{\mu\nu}$ violating the gauge group results in the equation:

$$D_\mu \widetilde{g}^{\mu\nu} = 0, \tag{10}$$

which is the consequence of the field equation and defines the polarization of the gravitational field (spin states 2 and 0) [12]. So, the structure of the mass part in the field equation and the field polarization are interdependent.

Now we have to consider an important consequence of the considered bimetric approach. The point is that the existence of the physically meaningful background spacetime imposes the *causality principle*, which constraints the permissible solutions in the RTG. This background defines the observable events and the corresponding relations between them. These relations always can be attributed to the Minkowski spacetime. Hence, the causality cone of the effective Riemannian spacetime should be positioned inside the causality cone of the Minkowski spacetime [13]:

$$\begin{aligned} \gamma_{\mu\nu}u^\mu u^\nu &= 0, \\ g_{\mu\nu}u^\mu u^\nu &\leq 0, \end{aligned} \tag{11}$$

where u^{μ} is the arbitrary isotropic vector.

The cosmological equations in the RTG can be obtained on the general basis. However, we have to take into account that the formally arbitrary choice of the convenient $g^{\mu\nu}$, which is a typical trick in the GR, is not always appropriate in the RTG because of it implies the simultaneous constraints on $\gamma_{\mu\nu}$.

Let us consider the homogeneous and isotropic Riemannian spacetime induced by the global gravitational field. As it was above mentioned, this spacetime in the framework of the RTG is *flat*. This is a consequence of the field equations (see [2, 3, 14]). The corresponding interval in the spherical coordinates is [3]:

$$ds^2 = d\tau^2 - \alpha a(\tau) \left[dr^2 + r^2 \left(d\theta^2 + \sin(\theta)^2 d\phi^2\right)\right], \tag{12}$$

where τ is the proper time, $a(\tau)$ is the scaling factor and α is the constant of integration (its meaning see below).

Let's the background is described by the Galilean metrics. Eqs. (12, 11) result in [3]:

$$a(\tau)^4 - \alpha < 0, \tag{13}$$

which eliminates the cosmological solution with the eternal expansion. This is a consequence of the causality principle in the RTG. It is convenient to assign $\alpha = a_{\max}^4$, where $a_{\max}$ is the maximum scaling factor.

Then the cosmological equations are:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho(\tau)}{3m_{pl}^2} - \frac{m^2}{12}\left(2 + \frac{1}{a(\tau)^6} - \frac{3}{a(\tau)^2 a_{\max}^4}\right), \tag{14}$$

$$\frac{\ddot{a}}{a} = -\frac{\rho(\tau) + 3p(\tau)}{6m_{pl}^2} - \frac{m^2}{6}\left(1 - \frac{1}{a(\tau)^6}\right), \tag{15}$$

and $a(\tau) \leq a_{\max}$. ρ and p are the matter density and pressure, respectively; the dot denotes the derivative with respect to τ. These equations are similar to those in the GR with the flat global spacetime but 1) to contain the terms describing the massive graviton and 2) to suppose the increase of a up to some maximum scaling factor $a_{\max}$ as a result of the causality principle.

3 Cosmological scenarios in the RTG and constraints on the graviton mass

Before an examination of the cosmological scenarios, let us consider the possible embedding of the effective Riemannian spacetime in the background with the constant curvature of the same dimension. The hyperbolic background has to be rejected due to the causality principle violation. The causally connected events in the effective spacetime are asymptotically causally independent on the background.

$$a^4 \leq \frac{\alpha}{1+r^2} \underset{r\to\infty}{\longrightarrow} 0. \tag{16}$$

However, the spherical background obeys the causality principle. The corresponding global Riemannian spacetime is spherical, too. Then the cosmological equations have the modified form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3m_{pl}^2} - \frac{m^2}{12}\left(2 + \frac{1}{a^6} - \frac{3}{a^2 a_{\max}^4}\right) - \frac{1}{a^2 a_{\max}^4}, \tag{17}$$

$$\frac{\ddot{a}}{a} = -\frac{\rho+3p}{6m_{pl}^2} - \frac{m^2}{6}\left(1 - \frac{1}{a^6} + \frac{3\left(1-\Sigma^2\right)}{4a^2 a_{\max}^4}\right), \tag{18}$$

where Σ is the background curvature, $a_{\max} = \Sigma$.

Turning back we can conclude that, although it is possible to embed the effective spacetime into the spherical background, there are no some physical justifications for such complication of the model. Nevertheless, the extension of the background dimensionality requires an additional analysis but this exceeds the limits of this article [15].

Let us return to Eqs. (14, 15) and consider their structure. It is clear that the fulfilment of the causality principle requiring only closed evolutional scenarios results from the first term in the brackets of Eq. (14). This term produced by the massive graviton plays a role of the negative cosmological constant, which stops any cosmological expansion of the universe with the arbitrary material filling if the state parameter for its dominating form is $w > -1$. The corresponding minimum density is connected with the graviton mass and the maximum scaling factor [3]:

Table 1: Cosmological parameters

Cosmological parameters	**Observational data**
$H_0,\ km\diagup(s\cdot Mps)$	68 ± 6
Ω_{tot}	1.11 ± 0.07
Ω_m	0.37 ± 0.07
Ω_r	$(9.34\pm 1.64)\times 10^{-5}$
Ω_x	0.71 ± 0.05
$\tau_0,\ Gyr$	12.7 ± 3
w	≤ -0.6
q	0.33 ± 0.17

$$\rho_{\min}=\frac{m^2 m_{pl}^2}{2}\left(1-\frac{1}{a_{\max}^6}\right). \tag{19}$$

On the other hand, the second term in the brackets of Eq. (15) causes the graviton mass induced repulsion (antigravitation) in the strong gravitational fields when the scaling factor is small. This repulsion prevents from the existence of the initial cosmological singularity and provides the acceleration at the initial stage of the universe expansion. However, as one can see from Eq. (15), out of this initial stage there is only decelerated expansion up to $a_{\max}$ if the state parameter for the dominating form of the matter is $w>-\frac{1}{3}$. We remind, that as a result of the vacuum stability principle (i.e. due to $g_{\mu\nu}\underset{T_{\mu\nu}\longrightarrow 0}{\longrightarrow}\gamma_{\mu\nu}$) the cosmological constant in the RTG has only gravitational nature and its sign is negative (i.e. it causes the attraction on the large scales). Here we face the challenge of the disagreement with the modern observational data.

The data obtained from the BOOMERANG, MAXIMA and COBE projects [6, 7, 16] suggest the accelerated expansion of the universe at present. The acceleration parameter can be estimated as $q\equiv\left(d^2a\diagup d\tau^2\right)|_0\diagup\left(a_0H_0^2\right)\simeq 0.33\pm 0.17$ and has a positive value (here H is the Habble constant and the zero index refers to the present epoch when $\tau=\tau_0$). On the whole the data are summarized in Table 1 (see also [17]).

The age of the universe τ_0 is estimated from the age of the oldest globular clusters. $\Omega_m\equiv\rho_m\diagup\left(3m_{pl}^2H_0^2\right)$ is the density parameter of the "normal" matter at present. The word "normal" means that this matter possesses the state parameter $w\geq 0$. Such matter can only decelerate the cosmological expansion. However, the detailed structure of this "normal" sector of the matter is unknown. The baryons contribution amounts only $\simeq 5\%$ in the total density and the rest of the matter sector belongs to the so-called cold dark matter, which is not revealed to day.

The similar parameter for the photons is $\Omega_\gamma \equiv \rho_\gamma \big/ \left(3m_{pl}^2 H_0^2\right) = 2.51 \times 10^{-5} h^{-2} \simeq (5.56 \pm 0.97) \times 10^{-5}$ (here $H_0 \equiv 100 \times h\ km \big/ (s \cdot Mps)$), and for the massless neutrino $\Omega_\nu = 0.681 \times \Omega_\gamma$ [18]. Then for the relativistic matter ("radiation") we have $\Omega_r \simeq (9.34 \pm 1.64) \times 10^{-5}$.

The accelerated expansion of the universe suggests that the main part of the density in the universe belongs to some exotic "dark energy" or "X-matter" with the density parameter Ω_x and the state parameter $w \leq -0.6$. The latter provides the negative pressure and, as a consequence, the acceleration of the universe expansion.

The parameter $\Omega_{tot} \equiv \Omega_x + \Omega_m + \Omega_r$ defines the curvature of the effective Riemannian spacetime in the GR through the so-called cosmic sum rule: $\Omega_{tot} + \Omega_K = 1$, where $\Omega_K \equiv -K \big/ \left(a_0^2 H_0^2\right)$ and $K = 1,\ -1$ and 0 for the spherical, hyperbolic and flat spacetimes, respectively. As $\Omega_{tot} \simeq 1$ at present [19], this requires the fine tuning at past (for example, if we start from the Planck scale the deviation from the unity at the beginning of the expansion is about of 10^{-60} [20]). There is no such problem in the RTG because the flatness of the homogeneous and isotropic Riemannian spacetime is the consequence of the field equations.

Now let us consider the possible modifications of the cosmological scenarios in the framework of the RTG, which provide the agreement with the modern observational data. To obtain the accelerated expansion at present we modify the energy-momentum tensor by the insertion of the quintessence term with the negative pressure. The practically interesting candidate for the quintessence is some scalar field ϕ, which evaluates slowly in a runaway potential $V(\phi)$: $V(\phi) \underset{\phi \longrightarrow \infty}{\longrightarrow} 0$ [21, 22, 23].

In the beginning let's consider the problem phenomenologically and introduce the quintessential term with the constant state parameter $w_x = p_x \big/ \rho_x$ lying in the limits of the strong and week energy conditions: $-1 < w < -1 \big/ 3$ [24, 25]. It is convenient to suppose that at present $a(\tau_0) = 1$ and to transit from the densities to the density parameters. Then Eq. (14) can be rewritten as:

$$H(\tau)^2 = H_0^2 \times \tag{20}$$
$$\left[\frac{\Omega_r}{a(\tau)^4} + \frac{\Omega_m}{a(\tau)^3} + \frac{\Omega_x}{a(\tau)^{(3+3w_x)}} - \Omega_g\left(1 + \frac{1}{2a(\tau)^6} - \frac{3}{2a(\tau)^2 a_{\max}^4}\right)\right],$$

where we used $\rho(\tau) \propto a(\tau)^{-3(1+w)}$ and introduced $\Omega_g = m^2 \big/ \left(6H_0^2\right)$ (the density parameter for the massive graviton).

We can see from Eq. (20) that the massive graviton modifies the cosmic sum rule

$$\Omega_r + \Omega_m + \Omega_x - \frac{3}{2}\Omega_g\left(1 - \frac{1}{a_{\max}^4}\right) = 1, \tag{21}$$

which is like that for the spherical curvature of the effective Riemannian spacetime ($a_{\max} \gg 1$). This similarity results from the negative cosmological constant-like action of the gravitons. Note however that in fact the spacetime is flat.

The substitution $t = H_0(\tau - \tau_0)$ in Eq. (15) produces (this substitution simply omits H_0^2 from the right-hand side of Eq. (20)):

$$\frac{d^2 a(t)}{dt^2} = -\frac{\Omega_m}{2a(t)^2} - \frac{\Omega_r}{a(t)^3} - \frac{1+3w_x}{2a(t)^{(2+3w_x)}} - \Omega_g\left(a(t) - \frac{1}{a(t)^5}\right). \tag{22}$$

Eqs. (20, 22) result in the expression for the acceleration parameter:

$$q = \frac{\Omega_x\left(1 - \frac{3}{2}\chi\right) - \frac{1}{2}\Omega_m - \Omega_r}{\Omega_{tot} - \frac{3}{2}\Omega_g}, \tag{23}$$

where $\chi \equiv 1 + w_x$ is the deviation of the quintessence state parameter from that for the pure positive cosmological constant. If the gravitons and the relativistic matter do not contribute to the present state, the combination of the observational data and Eq. (23) results in the estimation for χ:

$$\chi = \frac{2}{3}(1-q) - \frac{\Omega_m}{3\Omega_x}(1+2q) \simeq 0.16^{+0.11}_{-0.09}. \tag{24}$$

The deviation of the state parameter from that for the pure cosmological constant can be considered as the justification of the initial guess about the material (not vacuum) source of the accelerated expansion.

If $a_{\max} \gg a_0$ (this is a well-grounded assumption because the graviton mass has to be small, see below), then the minimum density is defined by the material terms with a slowest density decrease due to the scaling factor increase. These are the negative cosmological constant produced by the massive graviton and the quintessence with small χ. Hence we have the estimation for the maximum scaling factor:

$$\frac{\Omega_g}{\Omega_x} \simeq a_{\max}^{-3\chi}. \tag{25}$$

As a result, Eqs. (24, 25) give the dependence of the maximum scaling factor on the graviton density parameter. It is natural, the approach of w_x to -1 and Ω_x to 1 increase the maximum scaling factor due to growing negative pressure of the quintessence.

Eqs. (20, 25) allow to find the minimum scaling factor. The corresponding equation is:

$$\Omega_g a^{3w_x}\left(-2a^6 + \frac{3}{a_{\max}^4}a^4 - 1\right) + 2a^{3w_x}\left(\Omega_r a^2 + \Omega_m\right) + 2\Omega_x a^3 = 0. \tag{26}$$

If $w_x \gtrapprox -1$ and $a_{\max} \gg 1$ then $a_{\min} \simeq \sqrt{\Omega_g \big/ (2\Omega_r)}$. It is obviously that the minimum scaling factor can not be less than that corresponding to the radiation domination epoch. In this case we have the well-known condition (if the universe thermalized):

$$\rho_{\max} = \frac{\pi^2}{30} g_*\left(T\right) T_{\max}^4, \tag{27}$$

where $g_*(T)$ is the effective degeneracy factor, T is the temperature. Simultaneously, as the scaling factor is roughly $a_{\min} \simeq T_0/T_{\max}$ ($T_0 \simeq 10^{-4}\ eV$ is the present temperature of the cosmic background), we obtain the expression for the graviton density and the maximum scaling factor:

$$\begin{aligned} \Omega_g &\simeq 2\Omega_r \left(\frac{T_0}{T_{\max}}\right)^2, \\ a_{\max} &\simeq \sqrt[3\varkappa]{\left(\frac{\Omega_x}{2\Omega_r}\right)\left(\frac{T_{\max}}{T_0}\right)^2}. \end{aligned} \tag{28}$$

Table 2: Estimations of the maximum graviton mass and the maximum scaling factor. GUT is the grand unified theory phase transition, EW is the electroweak phase transition, NS is the nucleosynthesis, RD is the end of the radiation domination

Event	**T**	$\mathbf{m}_g,\ g$	$\mathbf{a}_{\max}$
GUT	$\sim 10^{15}\ GeV$	$\sim 10^{-94}\ \left(\sim 10^{-61}\ eV\right)$	10^{111}
EW	$\sim 100\ GeV$	$\sim 10^{-82}\ \left(\sim 10^{-49}\ eV\right)$	10^{53}
NS	$\sim 0.1\ MeV$	$\sim 10^{-76}\ \left(\sim 10^{-43}\ eV\right)$	10^{28}
RD	$\sim 1\ eV$	$\sim 10^{-71}\ \left(\sim 10^{-38}\ eV\right)$	10^{7}

The maximum admissible graviton mass and the maximum scaling factor are presented in Table 2 (for the cosmological parameters we choose their mean values). The estimation of the maximum graviton mass means that the universe starts its expansion from the denoted “event” (the dimensional mass can be re-calculated by means of the relation $m_g = \sqrt{6\Omega_g H_0 \hbar / c^2}$).

As it was above mentioned, the RTG solves some basic problems, which inspire the inflation paradigm in the modern cosmology: the flatness problem and the problem of the source of the initial expansion. Moreover, the inflation does not solve the problem of the singularity [26], which is lacking in the RTG. The problem of the relics in the RTG can be solved if $a_{\min}$ is too large to provide the sufficient for their creation $T_{\max}$. However, the problem of the horizon remains: the size of the causally connected domains at the moment of the last scattering of the cosmic background photons is ∼100 Mps. In principle, this problem can be solved without inflation (see, for example [27]). As the RTG eliminates singularity, it admits the physically meaningful oscillating solution with increasing homogeneity and isotropy. Nevertheless, let us examine the compatibility of the RTG with the inflation paradigm.

As it was mentioned, the feature of the RTG is the antigravitation produced by the massive graviton in the strong gravitational fields. This causes the accelerated expansion at the initial stage of the universe evolution and prevents from the singularity. However, the gravitational field is produced by the matter therefore the character of the initial acceleration is defined by the form of this matter. As an example, the relativist matter (radiation) results in the short acceleration stage (inflation) $t_{ac} = \Omega_g^{3/2}\left[1 - 1/\sqrt{2}\right] / \left(3\Omega_r^{5/2}\right)$ (t_{ac} is the acceleration time) with the scaling factor growing by only factor of root of two: $a_{end}/a_{\min} = \sqrt{2}$. It is clear that such short inflation is not sufficient for the solution of the horizon problem.

The appropriate choice is the inflation governed by the scalar field ϕ (inflaton). Let's consider the minimally coupled single scalar field with the potential $V(\phi)$. Then the initial evolution can be described by the following system:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_{pl}^2}\left(\frac{\dot{\phi}}{2} + V(\phi)\right) - \frac{m^2}{12a^6}, \tag{29}$$
$$\ddot{\phi} + 3\dot{\phi}\frac{\dot{a}}{a} = -\frac{dV}{d\phi}.$$

Note, that at the beginning there exist no other material fields with the exception of the scalar field. If at the beginning the potential energy prevails over the kinetic one, the exponential expansion begins (the standard slowroll conditions have to be satisfied: $m_{pl}\left|\frac{dV}{d\phi}\diagup V\right| \ll 1$ and $m_{pl}^2\left|\frac{d^2V}{d\phi^2}\diagup V\right| \ll 1$). The simple estimation shows that the graviton term vanishes very quickly and the expansion does not differ from that in the GR. The inflation ends when the sufficient energy transfers into the kinetic form. Then the so-called reheating begins and the material fields are created. The natural criterion providing this primordial inflation in the framework of the RTG is:

$$a_{\min} \leq a_{begin} \ll a_{end}, \tag{30}$$

where a_{begin} and a_{end} are the scaling factors at the beginning and at the end of the inflation, respectively.

Let us consider the potential, which admits the primordial inflation solving the horizon problem and, simultaneously, allows the second inflation describing the accelerated expansion at present. Such models consider both inflations as a manifestation of the single scalar field (quintessential inflation models, for review see [28]). For example, the potential [29]

$$\begin{aligned} V &= \lambda\left(\phi^4 + M^4\right) \text{ for } \phi < 0, \\ &= \frac{\lambda M^8}{\phi^4 + M^4} \qquad \text{for } \phi \geq 0 \end{aligned} \tag{31}$$

corresponds to the case of the self-interacting $\lambda\phi^4-$ field for the negative ϕ (the value of the cosmic background fluctuations requires $\lambda \leq 10^{-14}$) and provides the second acceleration on the rolling-away tail of the potential, when $\phi \longrightarrow \infty$ (quintessential tail). The present value of Ω_x requires $M \simeq 8 \times 10^5 \ GeV$.

The first inflation terminates at $|\phi| \sim m_{pl}$ and Eqs. (29, 30) result in $(M \ll m_{pl})$:

$$a_{end} \gg \sqrt[3]{\frac{m}{m_{pl}\sqrt{\lambda}}}. \tag{32}$$

Simultaneously,

$$\frac{a_r}{a_{end}} \simeq \frac{1}{\sqrt{\lambda R}}, \tag{33}$$

where $R \simeq 0.01$ is the numerical factor defining the particles creation at the end of the first inflation and the transit to the radiation domination [29]. At the beginning of the radiation domination, when $a \equiv a_r$, the temperature was

$$T_r \simeq \lambda R^{3/4} m_{pl} \simeq 10^3 \ GeV. \tag{34}$$

Then roughly we have:

$$m \ll \lambda^2 R^{3/2} m_{pl} \left(\frac{T_0}{T_r}\right)^3 \simeq 10^{-52} \ eV, \tag{35}$$

If we made the usual assumption about the 60-e folding expansion during the inflation then $a_{begin} \simeq a_{end} \times e^{-60}$. Hence we have the estimation for the graviton mass allowing the appropriate scaling factor:

$$m \sim \frac{\sqrt{12\lambda}}{m_{pl}} \phi_{in}^2 \left(a_{end} \times e^{-60}\right)^3 \simeq \frac{\sqrt{12}}{m_{pl}} \lambda^2 \Pi^{3/2} \phi_{in}^2 \left(\frac{T_0}{T_r}\right)^3 \times e^{-180} \tag{36}$$
$$\simeq 10^{-157} \frac{\phi_{in}^2}{m_{pl}},$$

where ϕ_{in} is the initial field and the graviton mass, which are expressed through the Planck mass. Although $|\phi_{in}| \gg m_{pl}$, the obtained estimation is extremely low because it is very hard to "squeeze" the universe down to the Planck scale in the condition of the strong antigravitation produced by the massive graviton. Such low value for the graviton mass can not be attributed to some real physics. However, we have not to consider this conclusion as the pessimistic estimation of the incompatibility between the RTG and the primordial inflation picture because 1) our estimation is the model-dependent and needs an additional investigation; 2) we have not to overestimate our knowledge of the physics on the Planck scale; 3) the RTG can propose an alternative (oscillating) scenario without primordial inflation.

It is of interest to consider the compatibility of the RTG with the second inflation picture, which takes a place on the quintessential tail of the model under consideration. In the framework of this model (see Eq. (31) (the below described picture is common for the different models of the quintessential inflation, see [28]) we have the following evolutional stages: 1) *First inflation.* The field $\phi \ll -m_{pl}$ slowly rolls to zero. The potential energy dominates over the kinetic one. As a consequence, the state parameter $w_x = \frac{\dot{\phi}^2/2-V}{\dot{\phi}^2/2+V} \approx -1$. 2) *Reheating and kination.* $\phi > -m_{pl}$ causes the end of the inflation due to the kinetic term increase ($w_x \longrightarrow 1$). The energy transfers to the material fields. But the kinetic dominated scalar field decreases as $\rho_x \approx \frac{\dot{\phi}^2}{2} \propto a^{-6}$. The radiation (and then the matter) domination begins. 3) The kinetic term vanishes and the potential energy of the scalar field dominates again. *Second inflation* begins from which the universe never recovers because the slowroll conditions are satisfied.

However, the RTG provides the quite natural exit from the eternal inflation due to the presence of the negative cosmological constant-like term in Eq. (17). As V decreases as [29]

$$V \sim \frac{\lambda M^8}{m_{pl}^2 \ln^4\left(\frac{a}{a_{end}}\right)}, \tag{37}$$

the inflation stops when

$$a \simeq a_{end} \exp\left(\frac{M^2\sqrt{m m_{pl}\sqrt{\lambda}}}{m m_{pl}^2}\right) \sim 10^{-24} \exp\left(\frac{10^{-14}}{\sqrt{m\,[eV]}}\right). \tag{38}$$

From Eq. (38) the maximum mass of the graviton is $\simeq 10^{-31}$ eV (criterion $a_{\max} > 1$), however, the maximum scaling factor increases exponentially with the m decrease in contrast to Eq. (25) because $w_x \approx -1$ in the late universe. Here we do not consider the additional numerical estimations because they are model-dependent. Nevertheless, it is obviously that the combination of the first inflation condition ($a_{\min} \ll a_{end}$) with the second inflation break can result in the exponentially large $a_{\max}$.

4 Conclusion

The RTG is able to solve some important cosmological problems. It does not contain the cosmological singularity and derives the flatness of the global spacetime from the field equations. The antigravitation produced by the massive graviton in the strong gravitational field solves the problem of the source of the initial expansion and allows to escape the relics creation. However, the problems of the horizon and the present accelerated expansion of the universe remain. The former is solvable in the framework of the oscillation paradigm. The RTG admits only closed evolutional scenario in virtue of the causality principle and thereby the causal connections with the extremely distant domains can result from the previous cycles of the oscillation (remind that there is no the singularity in the RTG). However, the problem of the accelerated expansion needs some additional hypothesis. The appropriate modification of the RTG Lagrangian is awkward because the structure of its massive part is defined by the polarization properties of the gravitational field. The alternative way is the modification of the energy-momentum tensor due to an inclusion of the so-called quintessence term with the state parameter lying between the limits of the strong and weak energy conditions that causes the repulsion and, as a result, the accelerated expansion.

In the framework of the latter approach there is the single scenario:

1. *First acceleration (inflation)*, which can be governed by the scalar field (exponential inflation) or by the massive gravitons (power-mode inflation that occurs if the radiation dominates at the beginning of the expansion).

2. *First deceleration* due to the radiation (and then matter) domination. The massive gravitons do not contribute due to the increased scaling factor.

3. *Second acceleration (inflation)* due to the quintessence domination. The massive gravitons do not contribute.

4. *Second deceleration* due to the negative cosmological constant-like action of the massive gravitons.

5. *Contraction* produced by the Massive gravitons.

At the first stage, there are the certain constraints on the graviton mass: the initial scaling factor has to provide the temperature, which is sufficient for the formation of the universe in its known form. At least, this temperature has to exceed that required for the nucleosynthesis. As a result, $m_g < 10^{-43}\ eV$. These constraints become extremely exacting in the case of the primordial inflation governed by the scalar field because it is hard to squeeze the universe down to the Planck size. One could say that the RTG is hardly compatible with this primordial inflation.

The second inflation can be considered in two ways. Firstly, we can suppose the constant state parameter for the quintessence: $w_x > -1$. In this case the massive graviton terminates the inflation for the scaling factor, which is power dependent on the graviton mass. The rough estimation for the above given m_g results in the relative scaling factor $\sim 10^{28}$ (if its present value is 1). Secondly, we can consider the quintessence as some scalar field with the rolling-away potential. This is a more complicated situation because the state parameter changes and approaches -1 in the late universe. However, the massive graviton stops the inflation in this case too, but the dependence of the maximum scaling factor on the graviton mass is exponential. As the latter approach is based on the artificial model building, the problem of the late universe evolution in the framework of the RTG needs an additional investigation.

In spite of the success in the agreement of the RTG with the modern observational data, the unsolved problems remain:

1. The horizon problem and the initial expansion of the universe remain unexplored in the RTG. There are some doubts about the compatibility of the RTG with the primordial inflation governed by the scalar field.

2. The quintessential scenarios need a more detailed investigation. Moreover, the nature of the quintessence is still unknown and this hypothesis faces some typical problems:

   (a) the quintessence has to be extremely weakly coupled with the usual matter;

   (b) it is probably that the quintessence has to be a very light [30];

(c) the quintessence would generate the corrections to the gauge coupling.

3. And at last, why is the graviton so light? It is necessary to explore the connections of the RTG with the modern field theory.

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